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Brief paper

Quantized-output feedback model reference control of discrete-time linear systems☆

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ABSTRACT

This paper proposes a model reference control (MRC) scheme for general discrete-time linear time-invariant (LTI) systems subject to output quantization and saturation. In such a scheme, the quantized-output feedback MRC law is analytically specified only by using external reference input and saturated-and-quantized output. It is proven that, by appropriately designing the sensitivity of the quantizer, the MRC law can ensure that all closed-loop signals are bounded and the output tracking error converges to a certain small residual set in a certain finite time only under the minimum-phase condition. Particularly, the proposed MRC scheme does not rely on the coprimeness of zero and pole polynomials or initial conditions that are commonly used in the related literature. A representative example is given to demonstrate the design procedure and verify the validity of the proposed MRC scheme.

1. Introduction

Over the past decades, we have witnessed a fast growing interest in how to effectively control linear and nonlinear systems subject to quantized and/or saturated measurements. One may refer to some overviews (Jiang & Liu, 2013; Tarbouriech & Turner, 2009). On one hand, traditional state or output feedback control methods are generically not able to be directly used for systems subject to quantized and/or saturated measurements. Thus, it is of importance to systematically develop quantized and/or saturated control theory. On the other hand, in real control systems, it often needs to overcome the restrictions of quantization and saturation problems, especially in networked control systems and digital control systems. Comparing to traditional control methods, quantized feedback control methods greatly reduce the requirement for sensor accuracy and magnitude. Moreover, in view of anti-noises, quantized feedback control has stronger robustness than exact feedback control. In a word, it is of greatly theoretical and practical importance to address quantized and/or saturated feedback control systems.

So far, quantized feedback control theory and applications have gained long-term developments, and a lot of remarkable results have been published. Quantized control research can be traced back to the 1960s (Larson, 1967). From then on, various quantized control methods have been published (Baillieul, 1999; Elia & Mitter, 2001; Fu & Xie, 2005; Gao & Chen, 2008; Hayakawa et al., 2009; Hespanha et al., 2002; Ishii & Francis, 2003; Li & Baillieul, 2004; Nair & Evans, 2000; Phat et al., 2004; Tatikonda & Mitter, 2004; Wong & Brockett, 1999; Yan et al., 2019). Quantized control idea was also widely used in multi-agent consensus or formation (Dimarogonas & Johansson, 2010; Franceschelli et al., 2011, 2014; Meng et al., 2016; Zhang & Zhang, 2013) and stochastic control systems (Liu et al., 2018; Wang et al., 2016a, 2016b). Considering that static quantization is difficult to realize global or semi-global convergence, Brockett and Liberzon (2000) for the first time proposed the dynamic quantization based control methodology. In Brockett and Liberzon (2000), the proposed methods achieve global or semi-global stabilization for LTI systems. Since then, dynamic quantization based stabilization problems have been systematically studied and consequently significant results were published (Fu & Xie, 2009; Liberzon, 2003; Liu et al., 2012; Moustakis et al., 2018). A

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systematic overview for stabilization of quantized feedback control systems has been given in Jiang and Liu (2013). While many quantized feedback control methods are focused on stability and stabilization problems, the quantized feedback tracking problem is of more practical meaning and has attracted a great deal of attention. In recent years, a lot of researchers turn to address the tracking control problems for systems with quantized measurement, and achieve significant progress (Bikas & Rovithakis, 2020; Fan et al., 2022; Liu et al., 2021; Lu & Liu, 2020; Ma et al., 2020; Sui & Tong, 2016; Yu et al., 2018; Yu & Lin, 2016; Zhou et al., 2018, 2013; Zouari et al., 2017). Quantized feedback control designs of nonlinear systems are mainly based on the well-known backstepping technique developed in Krstic et al. (1995). There are a vast number of related papers, and we are not able to list them all.

In recent years, under the foundation of standard MRC theory in Tao (2003), we addressed some problems yet to be solved in the literature, and published some results, for instance, Zhang et al. (2020, 2019, 2021). With our specialized experience, the remarkable result in Brockett and Liberzon (2000) motivates us to consider whether the quantized stabilizing method in Brockett and Liberzon (2000) can be extended to quantized-output feedback MRC control of discrete-time LTI systems. Consider a general class of discrete-time LTI systems described by $A(z)y(t) = k_pB(z)u(t)$, where y and u are the output and input, respectively, z is the forward shift operator, $k_p \neq 0$ is the high-frequency gain, and $A(z)$ and $B(z)$ are monic polynomials with constant coefficients and of degrees of n and m. It is well-known that the traditional MRC law is of the form $u(t) = \theta_1^{*T}\phi_1(t) + \theta_2^{*T}\phi_2(t) + \theta_3^{*}r(t)$, where $\theta_1^*, \theta_2^*, \theta_3^*$ are constant parameters of appropriate dimensions, $\phi_1(t), \phi_2(t)$ are well-defined signals, and $r(t)$ is an external reference input signal (Tao, 2003, 2014). Up to now, it is still unclear whether a saturated-and-quantized output feedback version of the traditional MRC law is still effective. Such a problem is never addressed before in the literature. In this paper, we for the first time give a positive answer. Based on the output feedback MRC theory in Tao (2003), we establish a basic quantized-output feedback MRC framework for discrete-time LTI systems only under the minimum-phase condition, i.e., the polynomial $B(z)$ is stable. In particular, the system observable or controllable condition, commonly used in existing quantized-output feedback control methods, is no longer needed in this paper. In summary, the contributions of this paper are as follows.

(i) A quantized-output feedback MRC scheme is developed for a general class of discrete-time LTI systems. We show that a quantized-and-saturated version of the standard MRC law is valid for control of the above systems.
(ii) In comparison with existing literature, the proposed MRC law has distinctive characteristics. First, only using external reference input and saturated-and-quantized output, the MRC law is analytically constructed. Second, only under the minimum phase condition, the MRC law can ensure that all closed-loop signals are bounded and the output tracking error converges to a certain residual set in a certain finite time. Last but not the least, the MRC law is independent of the system initial conditions.
(iii) The validity of the proposed MRC scheme is verified by a representative example with simulation results.

Notation: In the sequel, we use $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{Z}$ to denote the sets of real numbers, positive real numbers, and integers, respectively. We use z and z^{-1} to denote the forward and backward shift operators, i.e., $zx(t) = x(t+1)$ and $z^{-1}x(t) = x(t-1)$, where $t \in \{0, 1, 2, 3, \ldots\}$, $x(t) \triangleq x(tT)$ for a sampling period $T > 0$, and $x(t)$ denotes any signal of any finite dimension. We also use the notation L^∞ and $[\cdot]$: L^∞ denotes a signal space defined as $L^\infty = \{X(t) : \|X(\cdot)\|_\infty < \infty\}$ with $\|X(\cdot)\|_\infty \triangleq \sup_{t\geq 0}|X(t)|$; and $[\cdot]$ is defined as $[X(t)] \triangleq \max\{k \in \mathbb{Z} : k < X(t)\}$, where $X(t) \in \mathbb{R}$ denotes any signal on $\mathbb{R}$. We use the notation: $y(t) = G(z)u(t)$, to denote the output $y(t)$ of a discrete-time LTI system represented by a transfer function $G(z)$ with input $u(t)$. This notation is simple to combine both time and z-domain signal operations, helpful for control design and analysis, and useful to avoid causality contradiction problems and complex convolution expressions for control system presentation. Similar notation can be seen in Chen and Zhang (1990), Goodwin and Sin (1984), Tao (2003).

2. Problem statement

This section presents the system model and the problems to be addressed in this paper.

System model. Consider the following discrete-time single-input and single-output (SISO) LTI system:

$$A(z)y(t) = k_pB(z)u(t), \ \ t \geq t_0, \tag{1}$$

where t_0 is the initial time of the system operation, $k_p \neq 0$ is the constant high-frequency gain, and $A(z)$ and $B(z)$ are monic polynomials with constant coefficients and of degrees of n and m, respectively, i.e.,

$$A(z) = z^n + a_{n-1}z^{n-1} + \cdots + a_1z + a_0,$$
$$B(z) = z^m + b_{m-1}z^{m-1} + \cdots + b_1z + b_0.$$

Note that $n-m$ is the input–output delay, and also called the system relative degree (Tao, 2003). For the system model (1), we assume that: the values of $y(t)$ cannot be accurately measurable, and one can only acquire finite quantized values of $y(t)$, denoted as $q(y(t), \Delta(t))$, where $q : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{Z}$ is the quantizer, and $\Delta(t) > 0$ is a time-varying signal and called as the sensitivity of the quantizer (Brockett & Liberzon, 2000).

Dynamic quantizer. The quantizer in this paper is the same with that in Brockett and Liberzon (2000) and has the form

$$q(X(t), \Delta(t)) = \begin{cases} M, & \text{if } X(t) > (M+\frac{1}{2})\Delta(t), \\ \left[\frac{X(t)}{\Delta(t)} + \frac{1}{2}\right], & \text{if } -(M+\frac{1}{2})\Delta(t) < X(t) \leq (M+\frac{1}{2})\Delta(t), \\ -M, & \text{if } X(t) \leq -(M+\frac{1}{2})\Delta(t), \end{cases} \tag{2}$$

where $M \in \mathbb{Z}$ is a positive integer.

As stated in Brockett and Liberzon (2000), the form (2) of the quantizer has some certain physical meanings and potential applications, such as vision-based control. One may refer to Brockett and Liberzon (2000) to see a full clarification for the quantizer (2). The authors in Brockett and Liberzon (2000) proposed (2) to address the stabilization problem, while we use (2) in this paper to address the MRC problem that covers the stabilization problem as a special case.

Reference output model. The reference model is

$$y^*(t) = W_m(z)r(t), \ \ W_m(z) = \frac{1}{P_m(z)}, \tag{3}$$

where $P_m(z)$ is a stable monic polynomial of degree $n-m$ and $r(t) \in \mathbb{R}$ is an external reference input signal such that $r(t) \in L^\infty$. For discrete-time MRC, it is common to choose $P_m(z) = z^{n-m}$ so that

$$y^*(t) = r(t-n+m). \tag{4}$$

Control objective. For any given $y^*(t) \in L^\infty$, the control objective is to develop a quantized-output feedback control law $u(t)$ for the system (1) to ensure that closed-loop signals are bounded and

$y(t) - y^*(t)$ converges to a certain small residual set in a certain finite time.

Assumption. To meet the control objective, we only need the following assumption.

(A1): The polynomial $B(z)$ is stable.

Assumption (A1) requires the system (1) to be minimum-phase, which is needed for analyzing internal signal boundedness (Tao, 2003). Assumption (A1) indicates that the polynomials $A(z)$ and $B(z)$ are not required to be coprime, i.e., we allow common zeros and poles in the transfer function of the system (1).

Assumption (A1) is a consequence of zero-pole cancellations in MRC of LTI systems. The proposed MRC law will cancel the zeros of the system (1) and replaces them with those of the reference model. For stability, such cancellations should occur inside the unit circle of the complex z-plane, which implies that $B(z)$ should be stable. Similar explanations can be seen in Goodwin and Sin (1984), Ioannou and Sun (2012), Sastry and Bodson (1989), Tao (2003).

3. Fundamentals of output feedback MRC

We review some fundamentals of output feedback MRC of discrete-time LTI systems in Tao (2003), which will be used in quantized-output feedback MRC design.

Matching equation. Before giving the MRC law, we first present the following lemma which specifies a key equation for the control law design.

Lemma 1 (*Tao, 2003*)**.** *There exist constant vectors $\theta_1^* \in \mathbb{R}^{n-1}$ and $\theta_2^* \in \mathbb{R}^n$ such that*

$$\theta_1^{*T}\omega_1(z)A(z) + k_p\theta_2^{*T}\omega_2(z)B(z) = A(z) - B(z)z^{n-m}, \quad (5)$$

where $\omega_1(z)=[z^{-n+1}, \dots, z^{-1}]^T$ and $\omega_2(z)=[z^{-n+1}, \dots, z^{-1}, 1]^T$.

The proof of this lemma can be seen in Tao (2003). Eq. (5) is the well-known matching equation for output feedback MRC of LTI systems (Tao, 2003).

Output feedback MRC law. With θ_1^* and θ_2^* in (5), the MRC law is designed as

$$u(t) = \theta_1^{*T}\phi_1(t) + \theta_2^{*T}\phi_2(t) + \frac{1}{k_p}r(t), \ \ t \geq t_0, \quad (6)$$

with

$$\phi_1(t) = \omega_1(z)u(t), \ \ \phi_2(t) = \omega_2(z)y(t). \quad (7)$$

The following lemma specifies the MRC law capability.

Lemma 2 (*Tao, 2003*)**.** *There exist unique θ_1^* and θ_2^* that meet (5) and guarantee that the MRC law (6) which is applied to the system (1) leads to closed-loop stability and*

$$y(t+n-m) - y^*(t+n-m) = 0, \ \ \forall t \geq t_0. \quad (8)$$

The proof of Lemma 2 can be seen in Tao (2003). The parameters θ_1^* and θ_2^* in Lemma 2 are the so-called matching parameters because with these parameters the control law (6) leads to exact matching of the closed-loop system to the reference model (3).

Remark 3. Note that if $A(z)$ and $B(z)$ are not coprime, θ_1^* and θ_2^* satisfying (5) are not unique. However, Lemma 2 indicates that, no matter whether $A(z)$ and $B(z)$ are coprime or not, the parameters θ_1^* and θ_2^* in (6) are unique. This characteristic is proven in Tao (2003) and also can be concluded from the proof of Lemma 4 in the sequel. The proof of Lemma 4 also specifies how to determine the unique parameters θ_1^* and θ_2^* especially for the case when $A(z)$ and $B(z)$ are not coprime. □

Lemmas 1–2 are the fundamentals of MRC of discrete-time LTI systems and also the foundation of the quantized-output feedback MRC scheme that will be systematically addressed in the sequel.

4. Quantized-output feedback control design

Based on Lemmas 1–2, this section develops a quantized-output feedback MRC scheme for the system (1).

4.1. Quantized-output feedback MRC law structure

The standard MRC law (6) motivates us to design the quantized-output feedback MRC law of the structure

$$u(t) = \theta_1^{*T}\phi_1(t) + \theta_2^{*T}\phi_q(t) + \frac{1}{k_p}r(t), \ \ t \geq t_0, \quad (9)$$

where θ_1^*, θ_2^* are the unique parameters in Lemma 2, and

$$\phi_q(t) = \omega_2(z)(\Delta(t)q(y(t), \Delta(t)))$$

with $\Delta(t)$ to be designed later.

Tracking error equation. Define the quantized error and the tracking error as

$$s(y(t), \Delta(t)) = \Delta(t)q(y(t), \Delta(t)) - y(t),$$
$$e(t) = y(t) - y^*(t),$$

respectively. Now, we give the following lemma which specifies the tracking error equation that will be crucial for the sensitivity $\Delta(t)$ design and stability analysis.

Lemma 4. *The quantized-output MRC law (9), applied to the system (1), ensures*

$$e(t+n-m) = k_p\theta_2^{*T}\omega_2(z)s(y(t), \Delta(t)), \ \ \forall\, t \geq t_0. \quad (10)$$

Proof. From (5), we have

$$(\theta_1^{*T}\omega_1(z) - 1)A(z) = (-k_p\theta_2^{*T}\omega_2(z) - z^{n-m})B(z). \quad (11)$$

We first consider the case when $A(z)$ and $B(z)$ are coprime. It follows from (11) that, if z_i is a zero of $B(z)$, it must be a zero of $\theta_1^{*T}\omega_1(z) - 1$, otherwise (11) does not hold for $z = z_i$ with $B(z)$ and $A(z)$ being coprime. Thus, we conclude that there exists some polynomial

$$F(z) = -z^{-m} + f_{n-m-2}z^{-m-1} + \cdots + f_0 z^{-n+1}$$

with f_i, $i = 0, \dots, n-m-2$, being constant coefficients such that

$$F(z)B(z) = \theta_1^{*T}\omega_1(z) - 1. \quad (12)$$

In addition to (11), we obtain

$$k_p\theta_2^{*T}\omega_2(z) + F(z)A(z) = -z^{n-m}. \quad (13)$$

Then, operating both sides of (13) on $y(t)$ yields

$$k_p\theta_2^{*T}\omega_2(z)y(t) + F(z)A(z)y(t) = -y(t+n-m).$$

Together with (1), (7) and (12), we have

$$k_p\theta_2^{*T}\phi_2(t) + k_p(\theta_1^{*T}\omega_1(z) - 1)u(t) = -y(t+n-m).$$

After some manipulations with (4), (7), (9), we obtain

$$\begin{aligned}&y(t+n-m)-y^*(t+n-m)\\&=k_p\theta_2^{*T}(\phi_q(t)-\phi_2(t))=k_p\theta_2^{*T}\omega_2(z)s(y(t),\Delta(t)).\end{aligned}$$

Now we consider the case when $A(z)$ and $B(z)$ are not coprime. In this case, we rewrite $B(z)$ as $B(z)=B_1(z)B_2(z)$ such that $B_1(z)$ has degree n_1 and $B_2(z)$ and $A(z)$ are coprime. Then, there exist unique parameters $\bar{\theta}_1^*\in\mathbb{R}^{n-1-n_1}$ and $\theta_2^*\in\mathbb{R}^n$ such that

$$\begin{aligned}&z^{n-1-n_1}\bar{\theta}_1^{*T}\bar{\omega}_1(z)A(z)+k_p\theta_2^{*T}z^{n-1}\omega_2(z)B_2(z)\\&=z^{n-1-n_1}A(z)-z^{n-1}B_2(z)z^{n-m}\end{aligned}\tag{14}$$

with $\bar{\omega}_1(z)=[z^{-n+n_1+1},\dots,z^{-1}]^T$. With some manipulations, (14) becomes

$$\begin{aligned}&z^{-n_1}\bar{\theta}_1^{*T}\bar{\omega}_1(z)A(z)+k_p\theta_2^{*T}\omega_2(z)B_2(z)\\&=z^{-n_1}A(z)-B_2(z)z^{n-m}.\end{aligned}\tag{15}$$

Similar to (12), there exists some polynomial of the form

$$\begin{aligned}\bar{F}(z)=&-z^{-m+n_1}+\bar{f}_{n-m+n_1-2}z^{-m+n_1-1}\\&+\cdots+\bar{f}_0z^{-n+n_1+1}\end{aligned}$$

such that $\bar{F}(z)B_2(z)=\bar{\theta}_1^{*T}\bar{\omega}_1(z)-1$. Let $F(z)=z^{-n_1}\bar{F}(z)$. Then, in addition to (15), we also obtain (13), based on which the lemma's result follows. Note that, for the non-coprime case, the parameter θ_1^* is uniquely determined from the equation $\theta_1^{*T}\omega_1(z)-1=(\bar{\theta}_1^{*T}\omega_1(z)-1)B_1(z)z^{-n_1}=F(z)B(z)$. □

The tracking error equation (10) implies that exact output feedback, i.e., $s(y(t),\Delta(t))=0$, can achieve exact output tracking. However, for the quantized-output feedback case, before designing $\Delta(t)$, it cannot be sure whether s is bounded or not. Thus, the tracking error equation (10) does not imply the boundedness of e.

4.2. Technical lemmas

To proceed, we give the following two lemmas helpful to design $\Delta(t)$. Define

$$\lambda \triangleq \text{ an upper bound of } \max\{1+\text{magnitudes of }\lambda_i(A(z))\},\tag{16}$$

where $\lambda_i(A(z))$, $i=1,2,\dots,n$, denote the zeros of $A(z)$ on the complex z-plane.

Now, we give the following lemma.

Lemma 5. *For the system (1), if $u(t)=0$ and $\Delta(t)=c_0\lambda^{kt}$ with $c_0>0$ and $k>1$ being any two constants, then there always exists a well-defined number t_1 as*

$$t_1\triangleq\min\{t\ge t_0+1:|q(y(t),\Delta(t))|\le M-1\}.\tag{17}$$

Proof. If $u(t)=0$, $y(t)$ grows at most exponentially and $|y(t)|\le k_0(\lambda-1)^t$ with k_0 being some constant. If $\Delta(t)=c_0\lambda^{kt}$ with $c_0>0$ and $k\ge1$, $\Delta(t)$ grows faster than $|y(t)|$. Thus, no matter whether $q(y(t_0),\Delta(t_0))$ saturates or not, there exists some finite time instant t_p such that $q(y(t),\Delta(t))$ will never saturate for all $t\ge t_p$. Then, the lemma's result follows from the definition of q in (2). □

To design $\Delta(t)$, we also need the following lemma.

Lemma 6. *If $|y(t)|\le(M-\frac{1}{2})\Delta(t)$, then*

$$|s(y(t),\Delta(t))|\le\frac{1}{2}\Delta(t).$$

The proof of Lemma 6 is not difficult to perform. We omit it for space. Next, with Lemmas 5–6, we systematically address how to specify $\Delta(t)$.

4.3. Control design for systems with $n-m=1$

To show the basic ideas, we first consider the relative degree one case. Then, we address the general case.

Quantized-output MRC law for relative degree one case. For the system (1) with $n-m=1$, if there exists an integer $N\ge1$ such that

$$M-\frac{1}{2}\ge\frac{1}{\gamma^{N-1}}\left(c+1+\frac{d}{c_0\lambda^{kt_1}}\right)\tag{18}$$

with c_0,k,γ being constants such that $c_0>0$, $k>1$ and $0<\gamma<1$, and

$$\begin{aligned}&c\triangleq\text{ an upper bound of }\frac{1}{2}|k_p|\|\theta_2^*\|_1,\\&d\triangleq\text{ an upper bound of }|y^*(t)|,\end{aligned}\tag{19}$$

then the quantized-output feedback MRC law is designed as

$$u(t)=\begin{cases}0, & t\in[t_0,t_1),\\ \theta_1^{*T}\phi_1(t)+\frac{1}{k_p}r(t)\\ +\theta_2^{*T}\omega_2(z)(\Delta(t_i)q(y(t),\Delta(t_i))), & t\in[t_i,t_{i+1}),\\ & i=1,\dots,N-1,\\ \theta_1^{*T}\phi_1(t)+\frac{1}{k_p}r(t)\\ +\theta_2^{*T}\omega_2(z)(\Delta(t_N)q(y(t),\Delta(t_N))), & t\in[t_N,\infty),\end{cases}\tag{20}$$

where $\Delta(t)=c_0\lambda^{kt}$ for $t\in[t_0,t_1)$, and

$$\Delta(t_i)=c_0\gamma^{i-1}\lambda^{kt_1},\ \ i=1,2,\dots,N,\tag{21}$$

with λ in (16), t_1 in (17), and t_i, $i=2,3,\dots,N$, as

$$\begin{aligned}t_i\triangleq\min\Bigg\{&t\ge t_{i-1}+1:\\&|q(y(t),\Delta(t_{i-1}))|\le\frac{c\Delta(t_{i-1})+|y^*(t)|}{\Delta(t_{i-1})}+\frac{1}{2}\Bigg\}.\end{aligned}\tag{22}$$

System performance analysis. With the MRC law (20), we derive one of the main results as follows.

Theorem 7. *Under Assumption (A1), if the inequality (18) holds, then the quantized-output feedback MRC law (20), applied to the system (1) with $n-m=1$ and any unmeasurable $y(t_0)\in\mathbb{R}$, ensures that all closed-loop signals are bounded and the tracking error satisfies*

$$|e(t)|\le c_0c\lambda^{kt_1}\gamma^{N-1},\ \ \forall t\ge t_N+1.\tag{23}$$

Proof. For any unmeasurable $y(t_0)\in\mathbb{R}$, Lemma 5 ensures the existence of t_1. When $t=t_1$, from the definition of the quantizer in (2), we have $|y(t_1)|\le\Delta(t_1)\left(M-\frac{1}{2}\right)$. Moreover, when $t=t_1$, we change the control law from $u(t)=0$ to (20) with $\Delta(t)=\Delta(t_1)$. Then, from Lemma 4, we have

$$y(t_1+1)-y^*(t_1+1)=k_p\theta_2^{*T}\omega_2(z)s(y(t_1),\Delta(t_1)).\tag{24}$$

Since q does not saturate at $t=t_1$, Lemma 6 implies

$$|s(y(t_1),\Delta(t_1))|\le\frac{1}{2}\Delta(t_1).\tag{25}$$

Moreover, based on the fact that $\Delta(t)<\Delta(t_1)$ for all $t<t_1$, combining (24) and (25) yields

$$|y(t_1+1)|\le c\Delta(t_1)+|y^*(t_1+1)|$$

which follows from (18) that

$$|y(t_1+1)|\le\Delta(t_1)\left(M-\frac{1}{2}\right).$$

Thus, $q(y(t_1+1), \Delta(t_1))$ also does not saturate. Then, we can further verify that $q(y(t_1+2), \Delta(t_1))$ does not saturate, neither do $q(y(t_1+3), \Delta(t_1)), q(y(t_1+4), \Delta(t_1)), \ldots$. Therefore, we conclude that, if the control law is chosen as (20) with $\Delta(t) = \Delta(t_1)$ for $t \geq t_1$, $q(y(t), \Delta(t))$ will never saturate. Moreover, $e(t)$ satisfies that

$$|e(t)| \leq c\Delta(t_1), \ \forall t \geq t_1 + 1.$$

Thus, there exists a well-defined number t_2 in (22). Then, when $t = t_2$, we change the control law to (20) with $\Delta(t) = \Delta(t_2) = \gamma\Delta(t_1)$. Then,

$$|y(t_2)| \leq \Delta(t_2)\left(M - \frac{1}{2}\right)$$

which implies that $q(y(t_2), \Delta(t_2))$ does not saturate. Thus, using Lemma 4, we obtain

$$|y(t_2+1)| \leq c\Delta(t_2) + |y^*(t_2+1)|.$$

Together with (18), if $N \geq 2$, we have

$$|y(t_2+1)| \leq \Delta(t_2)\left(M - \frac{1}{2}\right)$$

which implies that $q(y(t_2+1), \Delta(t_2))$ does not saturate. Then, we can also verify that $q(y(t_2+2), \Delta(t_2))$ does not saturate, neither do $q(y(t_2+3), \Delta(t_2)), q(y(t_2+4), \Delta(t_2)), \ldots$. Therefore, we conclude that, if the control law is chosen as (20) with $\Delta(t) = \gamma\Delta(t_1)$ for $t \geq t_2$, $q(y, \Delta)$ will never saturate and $e(t)$ satisfies that

$$|e(t)| \leq \gamma c\Delta(t_1), \ \forall t \geq t_2 + 1.$$

Repeating the above procedure, we define $t_3, t_4, \ldots, t_N$ and obtain a sequence $\{\Delta(t_i)\}_{1\leq i\leq N}$. One can verify that the control law (20) ensures that $e(t)$ satisfies (23). By $y^*(t) \in L^\infty$, we have $y(t) \in L^\infty$. Under Assumption (A1), the boundedness of $y(t)$ implies that of $u(t)$. □

Based on (18) and (23), Theorem 7 indicates that if M is larger, N can be chosen larger and it follows from (23) that $e(t)$ can be made smaller.

Remark 8. For the sake of explanation, t_{i+1} in (22) can be roughly understood as the time when $e(t)$ first reaches the set $\{e(t) : |e(t)| \leq c\Delta(t_i)\}$. In the proof of Theorem 7, we have shown that the proposed MRC law (20) with $\Delta(t) = \Delta(t_i)$ and $t \geq t_i$ ensures that $e(t)$ reaches the above set in a finite time and remain inside thereafter. Note that we cannot use $e(t)$ to define t_i as $e(t)$ is not available. Instead, we use available signals $\Delta(t_i)$ and $y^*(t)$ to define t_{i+1} in (22). The finite time convergence of $e(t)$ guarantees that t_{i+1} in (22) is always existing and finite. □

4.4. Control design for systems with an arbitrary relative degree

Now, we give the following theorem which shows that the quantized-output MRC law (20) is also effective for the system (1) with $n - m > 1$.

Theorem 9. *Under Assumption (A1), if there exists an integer $N \geq 1$ such that*

$$M - \frac{1}{2} \geq \frac{1}{\gamma^{N-1}}\left(c + 1 + \frac{d}{c_0\lambda^{k(t_1+n-m-1)}}\right) \tag{26}$$

with c_0, k, γ, c, d being the same as before, then the quantized-output MRC law (20) with $\Delta(t) = c_0\lambda^{kt}$ for $t \in [t_0, t_1)$ and

$$\Delta(t_i) = c_0\gamma^{i-1}\lambda^{k(t_1+n-m-1)}, \ i = 1, 2, \ldots, N, \tag{27}$$

with t_1 in (17) and t_i in (22), applied to the system (1) with any unmeasurable $y(t_0) \in \mathbb{R}$ and $1 \leq n - m \leq n$, ensures that all closed-loop signals are bounded and

$$|e(t)| \leq c_0 c\lambda^{k(t_1+n-m-1)}\gamma^{N-1}, \ \forall t \geq t_N + n - m. \tag{28}$$

Proof. Similar to the $n - m = 1$ case, for any unmeasurable $y(t_0) \in \mathbb{R}$, Lemma 5 ensures the existence of t_1. When $t = t_1$, it follows the definition of the quantizer in (2) that

$$|y(t_1)| \leq c_0\lambda^{kt_1}\left(M - \frac{1}{2}\right).$$

For $t \geq t_1$, we change the control law from $u(t) = 0$ to (20) with $\Delta(t) = \Delta(t_1)$ in (27). In particular, when we change the control law at $t = t_1$, due to the input–output delay $n - m$, the control law (20) does not influence $y(t_1+j), j = 1, 2, \ldots, n-m-1$. Thus, based on the fact that $c_0\lambda^{kt}$ grows faster than $y(t)$, we have

$$|y(t_1+j)| \leq c_0\lambda^{k(t_1+j)}\left(M - \frac{1}{2}\right) \leq \Delta(t_1)\left(M - \frac{1}{2}\right) \tag{29}$$

for $j = 0, 1, \ldots, n-m-1$ and $\Delta(t_1) = c_0\lambda^{k(t_1+n-m-1)}$.

Then, (29) implies that $y(t_1+j), j = 0, 1, \ldots, n-m-1$, all do not saturate. Moreover, for $j = 0, 1, \ldots, n-m-1$, from Lemma 4, we have

$$y(t_1+n-m+j) - y^*(t_1+n-m+j)$$
$$= k_p\theta_2^{*T}\omega_2(z)s(y(t_1+j), \Delta(t_1)). \tag{30}$$

From (29) and Lemma 6, we obtain $|s(y(t_1+j), \Delta(t_1))| \leq \frac{1}{2}\Delta(t_1)$ for $j = 0, 1, \ldots, n-m-1$. Thus, together with (26) and (30), we have

$$|y(t_1+n-m+j)| \leq \Delta(t_1)\left(M - \frac{1}{2}\right)$$

which implies that $q(y(t_1+n-m+j), \Delta(t_1)), j = 0, 1, \ldots, n-m-1$, all do not saturate. Repeating the above procedure, we can further verify that $q(y(t_1+n-m+j), \Delta(t_1))$ for all $j > n-m-1$ do not saturate. Thus, we conclude that, if $\Delta(t)$ is chosen as $\Delta(t_1)$ in (27) for all $t \geq t_1$, q will never saturate and e satisfies that

$$|e(t)| \leq c\Delta(t_1), \ \forall t \geq t_1 + n - m.$$

Thus, there exists a well-defined number t_2 in (22). When $t = t_2$, we change the control law to (20) with $\Delta(t) = \Delta(t_2)$, where $\Delta(t_2)$ is defined in (27). Then, recalling the input–output delay $n - m$, we derive that $y(t_2+j), j = 0, 1, \ldots, n-m-1$, are still controlled by (20) with $\Delta(t) = \Delta(t_1)$. Thus, for $j = 0, 1, \ldots, n-m-1$, we have

$$|y(t_2+j)| \leq \gamma\Delta(t_1)\left(\frac{c}{\gamma} + \frac{|y^*(t_2+j)|}{\gamma\Delta(t_1)}\right)$$

which follows from (18) that

$$|y(t_2+j)| \leq \Delta(t_2)\left(M - \frac{1}{2}\right).$$

Thus, $q(y(t_2+j), \Delta(t_2)), j = 0, 1, \ldots, n-m-1$, all do not saturate, based on which we can further verify that $q(y(t_2+j), \Delta(t_2))$ do not saturate for all $j \geq n-m$. Hence, we conclude that, if the control law is chosen as (20) with $\Delta(t) = \Delta(t_2) = \gamma\Delta(t_1)$ for $t \geq t_2$, q will never saturate and e satisfies that

$$|e(t)| \leq \gamma c\Delta(t_1), \ \forall t \geq t_2 + n - m.$$

Repeating the above procedure, we define $t_3, t_4, \ldots, t_N$ of the form (22), and obtain a sequence $\{\Delta(t_i)\}_{1\leq i\leq N}$. Then, it can be verified that the control law (20) with $\Delta(t) = \Delta(t_i)$ that is defined in (27) ensures that $e(t)$ satisfies (28). Since $y^*(t) \in L^\infty$, we have $y(t) \in L^\infty$. Under Assumption (A1), the boundedness of $y(t)$ implies that of $u(t)$. □

So far, we have derived a complete quantized-output feedback MRC scheme for the system (1) with an arbitrary relative degree, which meets the control objective.

5. Simulation study

In this section, a representative example is given to illustrate the design procedure and the validity of the theoretical results.

Simulation model. Consider the following system

$$A_s(z)y(t) = k_{ps}B_s(z)u(t), \tag{31}$$

where $k_{ps} = 1$, and

$$A_s(z) = (z+1)(z-2)\left(z+\frac{1}{2}\right), \tag{32}$$

$$B_s(z) = z\left(z+\frac{1}{2}\right). \tag{33}$$

It follows from (32) and (33) that $A_s(z)$ is unstable and $B_s(z)$ is stable. Moreover, $A_s(z)$ and $B_s(z)$ have a common factor $z+\frac{1}{2}$ which is corresponding to the uncontrollable or unobservable mode of the state–space form.

Specification of θ_1^*, θ_2^* and $y^*(t)$. A key step for constructing the quantized-output feedback MRC law (20) is to specify $\theta_1^* \in \mathbb{R}^2$ and $\theta_2^* \in \mathbb{R}^3$. Note that θ_1^* and θ_2^* in (20) are the same with those in the standard output feedback MRC law (6). Thus, following the procedure of deriving θ_1^* and θ_2^* for standard output feedback MRC law of general discrete-time LTI systems in Tao (2003), we calculate θ_1^* and θ_2^* as

$$\theta_1^* = \left[0, -\frac{1}{2}\right]^T, \quad \theta_2^* = \left[-1, -\frac{5}{2}, -\frac{1}{2}\right]^T. \tag{34}$$

Moreover, $\phi_1(t)$ and $\phi_2(t)$ are specified as

$$\phi_1(t) = \omega_1(z)u(t), \quad \omega_1(z) = [z^{-2}, z^{-1}]^T, \tag{35}$$

$$\phi_2(t) = \omega_2(z)y(t), \quad \omega_2(z) = [z^{-2}, z^{-1}, 1]^T. \tag{36}$$

The reference output signal is chosen as

$$y^*(t) = \frac{1}{2}\sin(t) - \frac{1}{3}\cos(0.5t). \tag{37}$$

One can verify that exact output tracking can be achieved by applying the standard MRC law (6) with θ_1^* and θ_2^* in (34) and $\phi_1(t)$ and $\phi_2(t)$ in (35)–(36) to the system model (31).

Quantized-output feedback MRC law. From (16) and (32), we choose $\lambda = 3$. From (19) and (34), we obtain that $c = 4$. With (37), we choose $d = 1$. Based on (20) and (21), the constant parameters c_0, M, k are chosen as $c_0 = 1, M = 3\times 10^3 + 1, k = 1$. Moreover, we choose $\gamma = \frac{1}{2}$. Then, from (18), we determine $N = 13$. The control law in the simulation with $t_0 = 0$ is specified as

$$u(t) = \begin{cases} 0, & t \in [0, t_1), \\ \frac{1}{2}u(t-1) + r(t) + \Delta(t_i) \\ \cdot(-z^{-2} - \frac{5}{2}z^{-1} - \frac{1}{2})q(y(t), \Delta(t_i)), & t \in [t_i, t_{i+1}), \\ & i = 1, \dots, 12, \\ \frac{1}{2}u(t-1) + r(t) + \Delta(t_N) \\ \cdot(-z^{-2} - \frac{5}{2}z^{-1} - \frac{1}{2})q(y(t), \Delta(t_N)), & t \in [t_N, \infty), \end{cases} \tag{38}$$

where $\Delta(t) = 3^t$ for $t \in [0, t_1)$, and

$$r(t) = \frac{1}{2}\sin(t+1) - \frac{1}{3}\cos(0.5t + 0.5),$$

$$\Delta(t_i) = \frac{3^{t_1}}{2^{i-1}}, \quad i = 1, 2, \dots, 13, \tag{39}$$

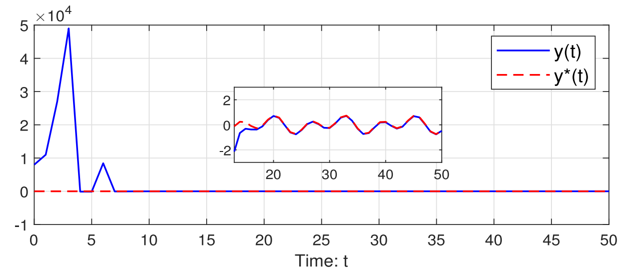

Fig. 1. Response of output $y(t)$ v.s. reference output $y^*(t)$.

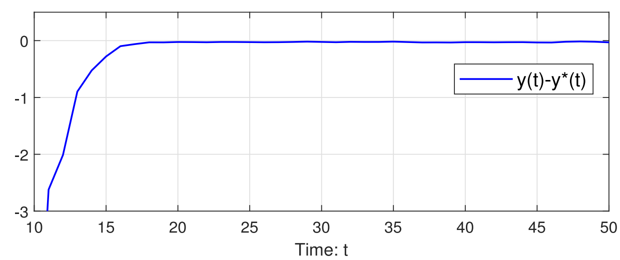

Fig. 2. Response of tracking error $y(t) - y^*(t)$.

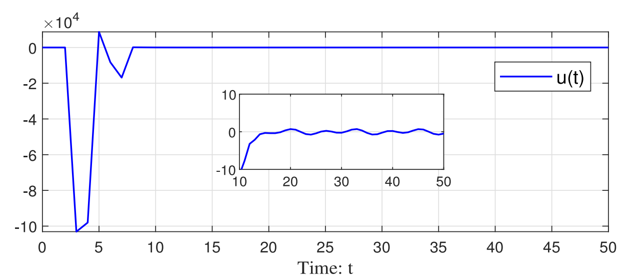

Fig. 3. Response of the quantized-output feedback MRC law (38).

with $t_1 = \min\left\{t \geq 1 : |q(y(t), \Delta(t))| \leq 3\times 10^3\right\}$ and t_i, $i = 2, \dots, 13$, can be derived from (22). Moreover, $q(y(t), \Delta(t))$ can be specified from (2).

Simulation results. To show the proposed control algorithm independent of the initial conditions, we choose $y(0) = 9000$ which is much larger than the saturated value M of the quantizer $q(y(t), \Delta(t))$.

Fig. 1 shows the response of the system output $y(t)$ versus the reference output $y^*(t)$ with Fig. 2 showing the tracking performance when $t \geq 10$. From Fig. 1 and Fig. 2, we see that satisfactory tracking is achieved when t is larger than about 18. Fig. 3 shows the response of the quantized-output feedback MRC law (38). Moreover, we present the response of the sensitivity $\Delta(t)$ in Fig. 4. In particular, the changes of t_i, $i = 1, 2, \dots, 13$, in (39) can be clearly obtained from Fig. 4. Specifically, we see from Fig. 4 that $t_1 = 3$, $t_2 = 6$, $t_3 = 8$, $t_4 = 9$, $t_5 = 10$, $t_6 = 11$, $t_7 = 12$, $t_8 = 13$, $t_9 = 14$, $t_{10} = 15$, $t_{11} = 16$, $t_{12} = 17$, and $t_{13} = 18$. By the way, $\Delta(t) = \frac{3^3}{2^{12}} = 0.00659$ for $t \geq 18$. Finally, we present the response of the quantizer $q(y(t), \Delta(t))$ in Fig. 5, in which $q(y(t), \Delta(t))$ is no longer saturated when $t \geq 3$. The evolution of $\Delta(t)$ and $q(y(t), \Delta(t))$ matches the theoretical results.

In summary, the simulation results for the system model (31) have not only verified the validity of the proposed method, but also verified the non-dependence on the initial conditions. Moreover, recalling that $A_s(z)$ and $B_s(z)$ are not coprime, the simulation results also verified the non-dependence of the proposed method on the coprime condition of zero and pole polynomials.

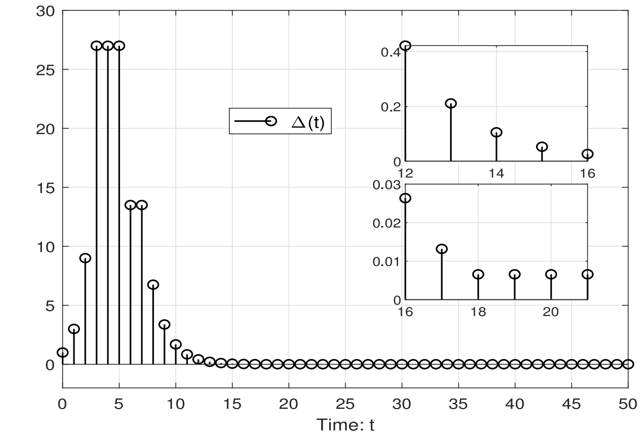

Fig. 4. Response of $\Delta(t)$.

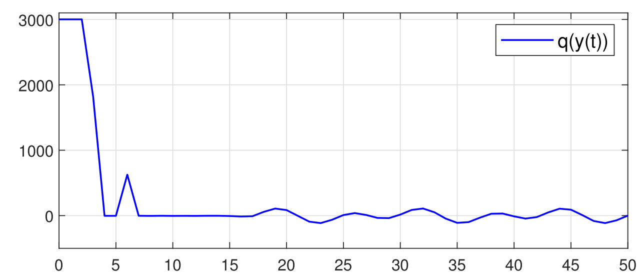

Fig. 5. Response of the quantizer $q(y(t), \Delta(t))$.

6. Concluding remarks

This paper has given a positive answer to the question: whether a quantized-output feedback version of the standard MRC law (20) is still effective for control of general discrete-time LTI systems without any additional design conditions. The quantized-output feedback MRC law is analytically constructed, and only replies on the minimum-phase condition.

It would be interesting to further consider the following problems: (i) if the coefficients of $A(z)$ and $B(z)$ are unknown, how to realize adaptive control based on the proposed control scheme in this paper? (ii) if $B(z)$ is not stable, that is, the system (1) is non-minimum phase, whether a quantized-output feedback version of the well-known pole placement method in Tao (2003) is still effective to achieve closed-loop stability and output tracking? (iii) whether the proposed method in this paper can be extended to the nonlinear systems case? These deserve further investigation.

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